Part I. (10.0 marks)

Questions 1 - 10 are short questions, each worth 1 mark, and you can answer without showing your working.

Question 1. Let \( \{x_n\} \) be a sequence given by

\[
\begin{align*}
x_1 &= \sqrt{6} \\
x_{n+1} &= \sqrt{6 + x_n}, n \geq 1
\end{align*}
\]

Find \([x_{2019}]\) (where \([x]\) is the Greatest Integer Function of \(x\)).

Question 2. For which values of \(m\), the equation

\[x^2 - (2m + 1)x + m^2 + 1 = 0\]

has two real solutions \(x_1, x_2\) such that \(x_1 = 2x_2\)?

Question 3. Suppose that \(x + y = 1\). Evaluate \(x^3 + y^3 + 3xy\).

Question 4. Solve the inequality \(3|2x - 1| < 2x + 1\).

Question 5. Evaluate \((4 + \sqrt{15})(\sqrt{10} - \sqrt{6})\sqrt{4 - \sqrt{15}}\).

Question 6. If \(2x^2 + 3y^2 \leq 5\), find the sum of the maximum value and the minimum value attained by \(2x + 3y\).

Question 7. \(n\) is the largest positive integer such that \(n^3 + 100\) is divisible by \(n + 10\). Find the digit sum of \(n\).

Question 8. Let \(a, b\) and \(c\) be real and positive parameters. How many solutions does the following equation have?

\[\sqrt{a + bx} + \sqrt{b + cx} + \sqrt{c + ax} = \sqrt{b - ax} + \sqrt{c - bx} + \sqrt{a - cx}.\]
Question 9. Let \( \{x_n\} \) be a sequence defined by

\[
\begin{cases}
x_0 = 3 \\
x_1 = 4 \\
x_{n+1} = x_{n-1}^2 - nx_n \forall n \geq 1.
\end{cases}
\]

Then \( x_{2019} = ? \)

Question 10. Given the real numbers \( a, b, c, d \) and \( e \) satisfy the relations \( a + b + c + d + e = 8 \) and \( a^2 + b^2 + c^2 + d^2 + e^2 = 16 \).

Determine the sum of the maximum value and the minimum value of \( a \).

Part II. (10.0 marks)

Questions 11 - 15 are longer questions, each worth 2 marks, and you have to show your working.

Question 11. Prove that \( \sin10^\circ \) is an irrational number.

Question 12. Consider a triangle \( \triangle ABC \), \( \angle BAC = 120^\circ \). Let \( AA_1, BB_1, CC_1 \) be three angle bisectors of \( \triangle ABC \) (\( A_1 \in BC, B_1 \in AC, C_1 \in AB \)). Prove that \( \angle B_1A_1C_1 = 90^\circ \).

Question 13. Determine the number of ways to choose 5 numbers from the first 18 positive integers such that any two chosen numbers differ by at least 2.

Question 14. Solve the equation

\[(x + 3)^3 - (x + 1)^3 = 56\]

Question 15. Prove that

\[16 < \sum_{k=1}^{80} \frac{1}{\sqrt{k}} < 17\]

The end.